REMARK. On again looking up our reference we find that we quoted the reference correctly, though as Mr. Heal observes, it seems that our reference is wrong. The value of the series

$$1-\frac{1}{3^{2n+1}}+\frac{1}{5^{2n+1}}-\frac{1}{7^{2n+1}}+\text{etc.}=\frac{\pi^3}{32}$$

is also given in Carr's *Synapsis of Pure Mathematics*, page 432. Since it now appears that the *value* of the integral is not known in finite terms, we will renew our offer to give the person finding such value a year's subscription to the MONTHLY. ED.

125. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Show that the *complete primitive* of the differential equation

$$\left[\tan^{-1}(x)-\frac{x}{1+x^2}\right]\frac{dy^2}{dx^2}=1\left[\frac{x}{(1+x^2)^2}\right]\left[x\frac{dy}{dx}-y\right]$$

is $y=C\tan^{-1}(x)+cx$.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

w being one solution of

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0\ldots\ldots(1),$$

and P, Q functions of x, $y=vw\ldots\ldots(2)$ is given, by the usual theory, from

$$\frac{d^2v}{dx^2}+\left(\frac{2}{w}\frac{dw}{dx}+P\right)\frac{dv}{dx}=0\ldots\ldots(3),\text{ or } v=c_2+c_1\int\frac{dx}{w^2}e^{-\int Pdx}\ldots\ldots(4).$$

In the given equation, $w=x\ldots\ldots(5)$, and

$$P=-\frac{2x^2}{(1+x^2)^2}\div\left[\tan^{-1}x-\frac{x}{1+x^2}\right]\ldots\ldots(6).$$

(4) now easily becomes

$$v=c_2+c_1\int\frac{dx}{x^2}\left(\tan^{-1}x-\frac{x}{1+x^2}\right)=c_2+c_1\left(\int\frac{\tan^{-1}x}{x^2}-\int\frac{dx}{x(1+x^2)}\right)$$

$$=c_2-c_1\frac{\tan^{-1}x}{x^2}\ldots\ldots(7),$$

and (2) is $y=c_2x-c_1\tan^{-1}x\ldots\ldots(8)$.

II. Solution by W. W. BEMAN. A. M., Professor of Mathematics, State University, Ann Arbor, Mich.

$$\left(\tan^{-1}x-\frac{x}{1+x^2}\right)\frac{d^2y}{dx^2}=\frac{2x}{(1+x^2)^2}\left(x\frac{dy}{dx}-y\right).$$

Writing the equation in the form

$$\frac{d}{dx}\left(\frac{\tan^{2}x}{x}\right)\cdot\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{1}{1+x^2}\right)\cdot\frac{d}{dx}\left(\frac{y}{x}\right)$$

it is obvious that $y=\tan^{-1}x$ and $y=x$ are independent particular integrals. Hence the complete primitive is $y=c_1\tan^{-1}x+c_2x$.

Solutions of this problem were also received from *L. C. WALKER*, and *G. B. M. ZERR*.

126. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find the volume contained between the conical surface whose equation is $z=a-\sqrt{(x^2+y^2)}$, and the planes whose equations are $x=z$ and $x=0$ by the formula $\int\int\int dxdydz$. [Todhunter's *Integral Calculus.*]

Solution by W. J. GREENSTREET. M. A., Editor of The Mathematical Gazette. Stroud. Gloucestershire, England.

The cone clearly extends from vertex (0, 0, a) towards $z=0$. Hence in

$$\int_{x_1}^{x_2}\int_{y_1}^{y_2}\int_{z_1}^{z_2} dxdydz$$

we have $z_1=x$; $z_2=a-\sqrt{(x^2-y^2)}$; $x=a-\sqrt{(x^2+y^2)}$.

$\therefore$ $y^2=a^2-2ax$, $x_1=0$, and for x_2 we have $y_1=y_2$. $\therefore$ $x=\frac{1}{2}a$.

$$\therefore \text{ Volume}=\int_0^{\frac{1}{2}a}\int_{y_1}^{y_2}[a-x-\sqrt{(x^2+y^2)}]dxdy$$

$$=\int_0^{\frac{1}{2}a}\left[(a-x)(y_2-y_1)-\frac{y}{2}\sqrt{(x^2+y^2)}-\frac{x^2}{2}\log y+\sqrt{(x^2+y^2)}\right]_{y_1}^{y_2}dx$$

$$=\int_0^{\frac{1}{2}a}\left[2(a-x)\sqrt{(a^2-2xa)}-\sqrt{(a^2-2xa)}(a-x)-\frac{x^2}{2}\log\frac{a-x+\sqrt{(a^2-2ax)}}{a-x-\sqrt{(a^2-2ax)}}\right]dx.$$

Put $2x=a\sin^2\varphi$.

$$\therefore\ V=\int_0^{\frac{1}{2}\pi}a\sin\phi\cos\phi d\phi\left[(1+\cos\phi)\frac{a^2}{2}\cos\phi-\frac{a^2}{8}\sin^4\phi\log\left(\frac{1+\cos\phi}{1-\cos\phi}\right)^2\right]$$

$$=\frac{a^3}{2}\int_0^{\frac{1}{2}\pi}\left[\sin\phi\cos^2\phi+\sin\phi\cos^4\phi-\sin^5\phi\cos\phi\log\frac{1+\cos\phi}{\sin\phi}\right]d\phi$$